Algebraic Proofs

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1) Algebraic Proofs: Easier

1) Prove that $\frac{1}{2}(n+1)(n+2) - \frac{1}{2}n(n+1) = n+1$

$= \frac{1}{2}(n+1)[n+2-n] \quad = \quad n+1$

$= \frac{1}{2}(n+1)[2] \quad = n+1$

$= \quad n+1 \quad = n+1$

LHS = RHS //

(2 Marks)

2) Prove that $(2n+1)^2 - (2n-1)^2$ is a multiple of 8 for all positive integer values of n.

$4n^2+4n+1 \quad - (4n^2-4n+1)$

$= 8n$

multiple of 8.

(2 Marks)

3) Prove that $(2n+1)^2 - (2n-1)^2 - 10$ is a not multiple of 8 for all positive integer values of n.

$= 4n^2+4n+1 \; - (4n^2-4n+1) - 10$

$= \quad 8n - 10$

$= \quad 8n \; - 8 - 2$

$\underbrace{8(n-1)} - 2$

divisible by 8, but not after -2

(2 Marks)

1) Algebraic Proofs: Medium

4) Prove algebraically that the sum of the squares of any two consecutive even numbers is always a multiple of 4.

$(2n)^2 + (2n+2)^2$

$4n^2 + 4n^2 + 8n + 4$

$8n^2 + 8n + 4$

$4(2n^2 + 2n + 1)$

$\rightarrow$

Always a multiple of 4

(2 Marks)

5) Prove algebraically that the difference between the squares of any two consecutive even numbers is always a multiple of 4.

$(2n+2)^2 - (2n)^2$

$4n^2 + 8n + 4 - 4n^2$

$8n + 4$

$4(2n+1)$

$\rightarrow$

Always a multiple of 4

(2 Marks)

6) Prove algebraically that the difference between the squares of any two consecutive numbers is always an odd number.

$(2n+1)^2 - (2n)^2$

$4n^2 + 4n + 1 - 4n^2$

$= 4n + 1$

$4n \rightarrow$ even

Adding 1 makes it odd

(2 Marks)

1) Algebraic Proofs: Harder

7) Prove algebraically that the sum of the squares of any two consecutive odd numbers cannot be a multiple of 4.

$(2n+1)^2 + (2n+3)^2$

$4n^2+4n+1 + 4n^2+12n+9$

$8n^2+16n+10$

$8n^2+16n+8+2$

$\underbrace{4(2n^2+4n+2)}_{\text{multiple of 4}}+2$

Adding 2 does not make it a multiple of 4 also

(2 Marks)

8) Prove algebraically that the sum of the squares of any three consecutive odd numbers always leaves a remainder of 11 when divided by 12.

$(2n+1)^2 + (2n+3)^2 + (2n+5)^2$

$4n^2+4n+1 + 4n+12n+9 + 4n^2+20n+25$

$12n^2+36n+35$

$12n^2+36n+24+11$

$\underbrace{12(n^2+3n+2)}_{\text{multiple of 12}}+11 \leftarrow$ Remainder of 11.

(2 Marks)